Quiz 4: April 19, 2019

Left Neighbor: ______________ Right Neighbor: ______________

Name: ______________ Student ID: ______________

Section TA: ______________

This is a closed book quiz

(You may not need this table, but it can't hurt, right?)

Name	**Equivalence**	
Identity Laws	$p \land \mathbf{T} \equiv p$	$p \lor \mathbf{F} \equiv p$
Domination Laws	$p \lor \mathbf{T} \equiv \mathbf{T}$	$p \land \mathbf{F} \equiv \mathbf{F}$
Idempotent Laws	$p \lor p \equiv p$	$p \land p \equiv p$
Double Negative Law	$\neg(\neg p) \equiv p$	
Commutative Laws	$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$
Associative Laws	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv q \land (p \land r)$
Distributive Laws	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
De Morgan's Laws	$\neg(p \land q) \equiv \neg p \lor \neg q$	$\neg(p \lor q) \equiv \neg p \land \neg q$
Absorption Laws	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
Negation Laws	$p \lor \neg p \equiv \mathbf{T}$	$p \land \neg p \equiv \mathbf{F}$
Def. of implication	$(p \rightarrow q) \equiv (\neg p \lor q)$	
Def. of equivalence	$\mathbf{p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p) \equiv (p \land q) \lor (\neg p \land \neg q)}$	

1. (5 points) Prove that if a and b are integers, and $a \mid b$, then $a^2 \mid b^2$.

$a \mid b \implies b = aX \ , X \in \mathbb{Z}$

$b^2 = a^2 \underbrace{X^2}_{C} = a^2 \cdot C \quad , C \in \mathbb{Z}$

$\implies b^2 = a^2 \cdot c \quad \implies a^2 \mid b^2$

2. (5 points) Prove that for Integers a, b, c, m, if $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$ then $a \equiv c \pmod{m}$.

$a \equiv b \pmod{m} \implies a = mq + b \ , q \in \mathbb{Z}$

$b \equiv c \pmod{m} \implies b = mf + c \ , f \in \mathbb{Z}$

Prove $a \equiv c \pmod{m} \implies a = ml + c \ , l \in \mathbb{Z}$

$a = mq + b = mq + mf + c$

$a = m\underbrace{(q+f)}_{n} + c \quad , n \in \mathbb{Z}$

$\implies a \equiv c \pmod{m}$